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Monterey, California



A Theorem of Anisotropic Absorbers

by

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March 1997

DTIC QUALITY INSPECTED 4

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Prepared for: Naval Postgraduate School
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19970422 124

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REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.

1. AGENCY USE ONLY (<i>Leave blank</i>)		2. REPORT DATE March 1997		3. REPORT TYPE AND DATES COVERED Technical Report	
4. TITLE AND SUBTITLE A Theorem of Anisotropic Absorbers			5. FUNDING NUMBERS N/A		
6. AUTHOR(S) Hung-Mou Lee and Chen-Kuo Yu					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey CA 93943-5000			8. PERFORMING ORGANIZATION REPORT NUMBER NPS-EC-97-004		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey CA 93943-5000			10. SPONSORING/MONITORING AGENCY REPORT NUMBER		
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.					
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE		
13. ABSTRACT (<i>maximum 200 words</i>) The sum-difference surface current formulation is introduced to treat electromagnetic boundary value problems when anisotropic impedances are specified on both sides of a surface. It can also be applied to impedance coated bodies. This formulation preserves the duality nature of Maxwell equations and carries it over into the algebraic form of the integrodifferential operators in the equations for surface currents. Since a 90° rotation is equivalent to undergoing a duality transform for an incident plane wave, this particular symmetry in the algebraic form of the operators leads to sufficient conditions under which the on-axis backscattering of an anisotropic impedance coated scatterer having a 90° rotational symmetry is eliminated.					
14. SUBJECT TERMS zero backscattering cross section, impedance boundary condition, anisotropic surface impedance				15. NUMBER OF PAGES 36	
17. SECURITY CLASSIFI- CATION OF REPORT Unclassified		18. SECURITY CLASSIFI- CATION OF THIS PAGE Unclassified		16. PRICE CODE 19. SECURITY CLASSIFICA- TION OF ABSTRACT Unclassified	
20. LIMITATION OF ABSTRACT unlimited					

Abstract

The sum-difference surface current formulation is introduced to treat electromagnetic boundary value problems when anisotropic impedances are specified on both sides of a surface. It can also be applied to impedance coated bodies. This formulation preserves the duality nature of Maxwell equations and carries it over into the algebraic form of the integrodifferential operators in the equations for surface currents. Since a 90° rotation is equivalent to undergoing a duality transform for an incident plane wave, this particular symmetry in the algebraic form of the operators leads to sufficient conditions under which the on-axis backscattering of an anisotropic impedance coated scatterer having a 90° rotational symmetry is eliminated.

TABLE OF CONTENTS

I. INTRODUCTION	1
II. THE SUM-DIFFERENCE SURFACE CURRENT FORMULATION OF ELECTROMAGNETIC BOUNDARY-VALUE PROBLEMS	5
A. STRATTON-CHU FIELD FORMULATION AND RADIATION	5
B. CONDITION ON THE CURRENTS IMPOSED BY MAXWELL EQUATIONS	7
C. IMPEDANCE BOUNDARY CONDITION	9
D. ALGEBRA OF THE SUM-DIFFERENCE CURRENT EQUATIONS	11
E. CONSIDERATIONS FOR A CLOSED SURFACE	14
III. A THEOREM OF ANISOTROPIC ABSORBERS	15
A. AXIAL RADIATION FROM A SURFACE OF 90° ROTATIONAL SYMMETRY	15
B. CONDITION FOR VANISHING ON-AXIS BACKSCATTERING ..	17
C. IMPEDANCE MATRICES FOR ZERO ON-AXIS BACKSCATTERING	21
IV. CONCLUSIONS	25

LIST OF REFERENCES 27

INITIAL DISTRIBUTION LIST 29

I. INTRODUCTION

Sometime ago the question was raised: "For electromagnetic boundary value problems with specified surface impedances, how can one go from a non-perfectly conducting surface on which both the electric and the magnetic equivalent surface currents are to be found, to a perfectly conducting surface on which the number of unknowns is halved [1]?" The answer to this question turns out to be one of algebra. It is well known that the impedance specified on the surface of a body separates its interior completely from its exterior. Therefore an impedance coated body can always be considered as a hollow volume enclosed by an infinitesimally thin shell with surface impedances specified both on the inside and the outside of the shell. The inside and the outside of the body can be considered as constituted of the same medium and the impressed electromagnetic excitation can be treated as continuous across the shell. On the outside surface, there are the equivalent total electric current \vec{K}^+ and total magnetic current \vec{L}^+ ; on the inside surface, there are \vec{K}^- and \vec{L}^- . For an exterior problem, only \vec{K}^+ and \vec{L}^+ need to be found; for an interior problem, only \vec{K}^- and \vec{L}^- are necessary. A single formulation for solving both types of problems would have required finding all inside and outside currents therefore doubling the amount of work, but it turns out not to be the case because some of the currents are linear combinations of others. Furthermore, this formulation holds the key to answering the question posed above.

Since the shell is infinitesimally thin, from Maxwell equations the radiation to the outside and to the inside of the shell can both be given in terms of integrodifferential operators on the sum currents $\vec{K} = \vec{K}^+ + \vec{K}^-$ and $\vec{L} = \vec{L}^+ + \vec{L}^-$. Note that the outside currents

will not contribute to the radiation in the interior while the inside currents will not contribute to the radiation to the exterior of the shell. For simplicity in the description, we consider the exterior problem of electromagnetic scattering. By definition, the radiation is the difference between the total field and the incident field. On the surfaces of the shell, this definition links the incident \vec{E} and \vec{H} fields and the difference currents $\vec{K}^+ - \vec{K}^-$ and $\vec{L}^+ - \vec{L}^-$ to the sum currents. It is therefore natural to treat the difference currents and the sum currents as the four unknowns to be solved instead of the inside and the outside currents.

The surface impedance on the outside surface of the shell normalized to the medium is denoted by Z^+ and that on the inside surface is Z^- . They can be tensors if the impedances are anisotropic and may vary from point to point. By forming the sum impedance $Z = (Z^+ + Z^-)/2$ and the difference impedance $\Delta = (Z^+ - Z^-)/2$, the impedance boundary conditions provide a set of relations between the difference and the sum currents. It turns out that the rank of Z determines how the unknown surface currents are solved. If Z is invertible, then the difference currents are linear combinations of the sum currents so that only the integrodifferential equations of the sum currents have to be solved. There are only two unknowns to be solved for both the exterior and the interior problems under this sum-difference current formulation. If Z is rank 0, then $Z = 0$; the impedance boundary condition requires that \vec{L} be proportional to a 90° rotation of $\Delta \vec{K}$. The difference electric current is obtained from \vec{K} after the integrodifferential equation on \vec{K} is solved. This situation includes the case where $Z^+ = Z^- = 0$ when the surface is perfectly conducting, thus the result answers the question about the transition of the equations from a problem of two variables to one

which has only a single variable.

Instead of dealing with an impedance coated body, this report presents the sum-difference currents formulation of electromagnetic boundary value problems for the scattering of an infinitesimally thin surface for which both the inside and the outside currents are true unknowns to be found. Extension of this formulation to impedance coated bodies is then discussed.

This formulation preserves the duality nature of Maxwell equations and carries it over into an explicit specific algebraic form of the integrodifferential operators in the equations for the sum currents. Since, for a plane wave, a 90° rotation is equivalent to undergoing a duality transform, this explicit symmetry in the algebraic form of the operators enables us to deduce sufficient conditions for eliminating the on-axis backscattering of an anisotropic impedance coated scatterer having a 90° rotational symmetry.

In this report, the time dependence $e^{-i\omega t}$ is used. \vec{E} represents the electric field intensity divided by the intrinsic impedance of the medium, $\sqrt{\mu/\epsilon}$; therefore it has the same unit as \vec{H} in amperes per meter. So are the electric and magnetic equivalent surface currents \vec{K} and \vec{L} .

II. THE SUM-DIFFERENCE SURFACE CURRENT FORMULATION OF ELECTROMAGNETIC BOUNDARY-VALUE PROBLEMS

A. STRATTON-CHU FIELD FORMULATION AND RADIATION

On an orientable, piecewise regular surface, whether open or closed, having a surface electric current density \vec{K} and a surface magnetic current density \vec{L} , we define the Stratton-Chu E-field formula as:

$$\begin{aligned}\vec{E}_{s-c}(\vec{r}, S, \vec{K}, \vec{L}) &= \frac{ik^2}{4\pi} \int_S \vec{K}(\vec{r}_o) G(\vec{r}-\vec{r}_o) da_o - \frac{i}{4\pi} \nabla \int_S \vec{K}(\vec{r}_o) \cdot \nabla_o G(\vec{r}-\vec{r}_o) da_o \\ &\quad - \frac{k}{4\pi} \nabla \times \int_S \vec{L}(\vec{r}_o) G(\vec{r}-\vec{r}_o) da_o\end{aligned}\quad (2-1)$$

where \vec{r} is a point which is not on S , $k = \omega\sqrt{\mu_o \epsilon_o}$ and $G(\vec{r}-\vec{r}_o) = \frac{e^{ik|\vec{r}-\vec{r}_o|}}{k|\vec{r}-\vec{r}_o|}$, then the Stratton-Chu H-field formula can be defined as:

$$\begin{aligned}\vec{H}_{s-c}(\vec{r}, S, \vec{K}, \vec{L}) &= \vec{E}_{s-c}(\vec{r}, S, \vec{L}, -\vec{K}) \\ &= \frac{k}{4\pi} \nabla \times \int_S \vec{K}(\vec{r}_o) G(\vec{r}-\vec{r}_o) da_o + \frac{ik^2}{4\pi} \int_S \vec{L}(\vec{r}_o) G(\vec{r}-\vec{r}_o) da_o \\ &\quad - \frac{i}{4\pi} \nabla \int_S \vec{L}(\vec{r}_o) \cdot \nabla_o G(\vec{r}-\vec{r}_o) da_o\end{aligned}\quad (2-2)$$

Note that if S is a closed surface and \vec{K} and \vec{L} are the actual total equivalent surface currents on S , then \vec{E}_{s-c} and \vec{H}_{s-c} are respectively the \vec{E} and \vec{H} fields at \vec{r} due to all sources inside S if \vec{r} is located outside S and vice versa. This is a direct consequence of Maxwell equations [2] and under this circumstance, the Stratton-Chu formulae are equivalent to Maxwell equations. On the other hand, unlike Poynting theorem, the Stratton-Chu field formulae over an open surface S are but integrodifferential operators on the tangential vector fields \vec{K} and \vec{L} .

over S without any special physical meaning attached.

To introduce equivalent surface currents on S , the direction of the unit normal vector \hat{n} on the surface has to be chosen. Adopting the terminology for a closed surface, we can assign one side of any orientable surface S as the "outside surface" S^+ , albeit somewhat arbitrarily if S is not closed. The outward normal \hat{n}^+ is the unit normal pointing out of this side of S and for simplicity, we call \hat{n}^+ the normal of S and denote it by \hat{n} . The other side of the surface S is the "inside surface" S^- . At any point \vec{r} on S , the inward normal \hat{n}^- is $-\hat{n}$. As a convention, the fields and surface currents on S^+ and S^- always carry the corresponding superscripts (Figure 2-1).

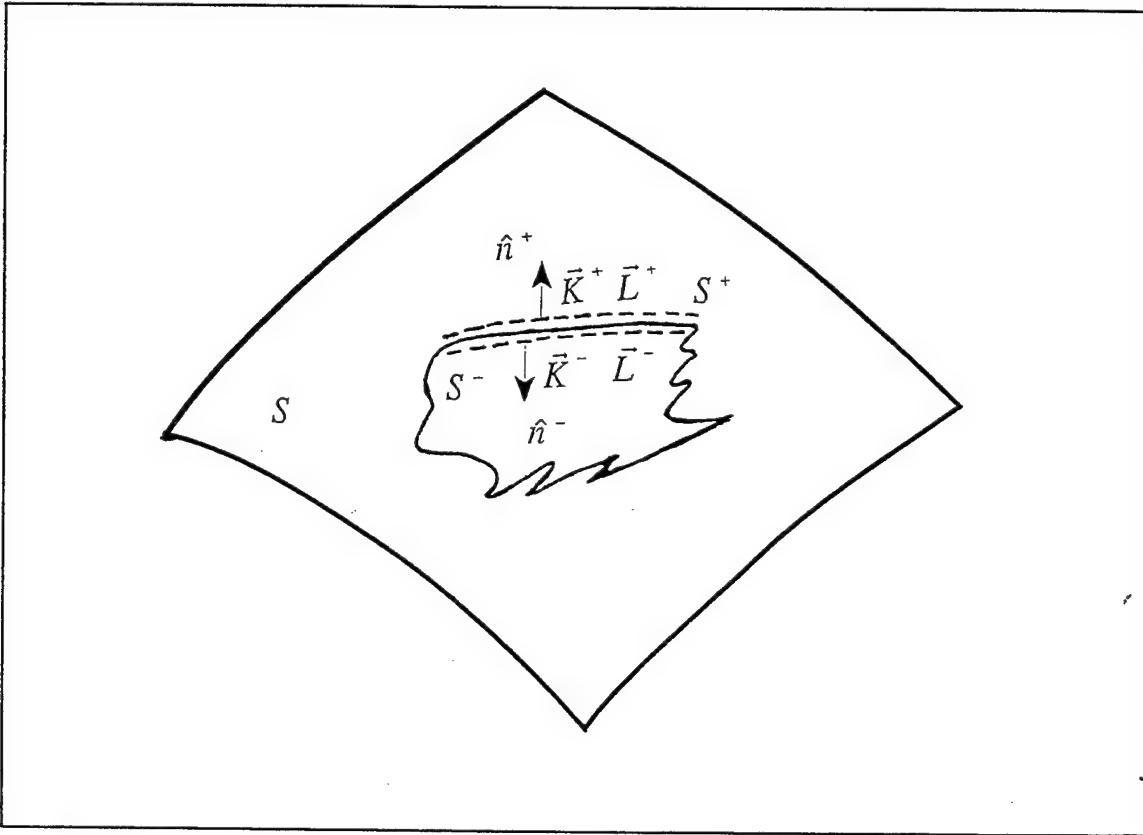


Figure 2-1. Outside and Inside Surfaces, Normals and the Equivalent Currents.

Each of the total surface currents \vec{K}^\pm and \vec{L}^\pm on S^\pm consists of two parts: the incident

current (with the additional superscript "inc") and the scattering current (with the additional superscript "sc") corresponding to the incident field and the scattered field on the particular side of the surface S:

$$\begin{aligned}\vec{K}^{\pm} &= \hat{n}^{\pm} \times \vec{H}^{\pm} = \hat{n}^{\pm} \times \vec{H}^{inc} + \hat{n}^{\pm} \times \vec{H}^{\pm,sc} \\ &= \vec{K}^{\pm,inc} + \vec{K}^{\pm,sc}\end{aligned}\quad (2-3)$$

$$\begin{aligned}\vec{L}^{\pm} &= \vec{E}^{\pm} \times \hat{n}^{\pm} = \vec{E}^{inc} \times \hat{n}^{\pm} + \vec{E}^{\pm,sc} \times \hat{n}^{\pm} \\ &= \vec{L}^{\pm,inc} + \vec{L}^{\pm,sc}\end{aligned}\quad (2-4)$$

Note that S is infinitesimally thin, hence $\vec{H}^{+,inc} = \vec{H}^{-,inc} = \vec{H}^{inc}$ and $\vec{E}^{+,inc} = \vec{E}^{-,inc} = \vec{E}^{inc}$ on S so that $\vec{K}^{+,inc} = -\vec{K}^{-,inc}$ and $\vec{L}^{+,inc} = -\vec{L}^{-,inc}$. Therefore the sum currents \vec{K} and \vec{L} on S defined below are also the corresponding sums of the scattering currents only:

$$\begin{aligned}\vec{K} &= \vec{K}^+ + \vec{K}^- = \vec{K}^{+,sc} + \vec{K}^{-,sc} \\ \vec{L} &= \vec{L}^+ + \vec{L}^- = \vec{L}^{+,sc} + \vec{L}^{-,sc}\end{aligned}\quad (2-5)$$

Since the Stratton-Chu field formulae are linear operators on the surface currents, the radiated fields from surface currents on S are determined by the sum currents only:

$$\begin{aligned}\vec{E}^{sc}(\vec{r}) &= \vec{E}_{s-c}(\vec{r}, S^+, \vec{K}^+, \vec{L}^+) + \vec{E}_{s-c}(\vec{r}, S^-, \vec{K}^-, \vec{L}^-) = \vec{E}_{s-c}(\vec{r}, S, \vec{K}, \vec{L}) \\ \vec{H}^{sc}(\vec{r}) &= \vec{H}_{s-c}(\vec{r}, S, \vec{K}, \vec{L}) = \vec{E}_{s-c}(\vec{r}, S, \vec{L}, -\vec{K})\end{aligned}\quad (2-6)$$

B. CONDITION ON THE CURRENTS IMPOSED BY MAXWELL EQUATIONS

As \vec{r} approaches \vec{r}^{\pm} on S^{\pm} , the tangential components (denoted by the subscript "tan") of Eq. (2-6) provide four equations relating the incident fields and the total currents

on both sides of S through the fact that the incident field is the difference between the total and the scattered field:

$$\vec{E}_{\tan}^{inc} = \vec{E}_{\tan}^+ - \vec{E}_{s-c,\tan}(\vec{r}^+, S, \vec{K}, \vec{L}) \quad (2-7)$$

$$\vec{E}_{\tan}^{inc} = \vec{E}_{\tan}^- - \vec{E}_{s-c,\tan}(\vec{r}^-, S, \vec{K}, \vec{L}) \quad (2-8)$$

$$\vec{H}_{\tan}^{inc} = \vec{H}_{\tan}^+ - \vec{H}_{s-c,\tan}(\vec{r}^+, S, \vec{K}, \vec{L}) \quad (2-9)$$

$$\vec{H}_{\tan}^{inc} = \vec{H}_{\tan}^- - \vec{H}_{s-c,\tan}(\vec{r}^-, S, \vec{K}, \vec{L}) \quad (2-10)$$

These four equations are not independent of each other. Because

$$\begin{aligned} \hat{n}^\pm \times (\vec{E}(\vec{r}^\pm) \times \hat{n}^\pm) &= \vec{E}_{\tan}(\vec{r}^\pm) = \hat{n}^\pm \times \vec{L}^\pm \\ \hat{n}^\pm \times (\vec{H}(\vec{r}^\pm) \times \hat{n}^\pm) &= \vec{H}_{\tan}(\vec{r}^\pm) = -\hat{n}^\pm \times \vec{K}^\pm \end{aligned} \quad (2-11)$$

the difference between Eqs. (2-7) and (2-8) trivially confirms the definition of the sum equivalent magnetic current while the difference between Eqs. (2-9) and (2-10) confirms the definition of the sum equivalent electric current as both can be deduced directly from Maxwell equations. We choose to use the sum of Eqs. (2-7) and (2-8) and that of Eqs. (2-9) and (2-10) as the two independent linear combinations out of Eqs. (2-7) through (2-10) to link the incident fields to the total surface currents on S^\pm as dictated by Maxwell equations:

$$\begin{aligned}
2\vec{E}_{\tan}^{inc} &= \hat{n} \times (\vec{L}^+ - \vec{L}^-) - \left\{ \vec{E}_{s-c}(\vec{r}^+, S, \vec{K}, \vec{L}) + \vec{E}_{s-c}(\vec{r}^-, S, \vec{K}, \vec{L}) \right\} \\
&= \hat{n} \times (\vec{L}^+ - \vec{L}^-) + M\vec{K} - N\vec{L} \\
2\vec{H}_{\tan}^{inc} &= -\hat{n} \times (\vec{K}^+ - \vec{K}^-) - \left\{ \vec{H}_{s-c}(\vec{r}^+, S, \vec{K}, \vec{L}) + \vec{H}_{s-c}(\vec{r}^-, S, \vec{K}, \vec{L}) \right\} \\
&= -\hat{n} \times (\vec{K}^+ - \vec{K}^-) + N\vec{K} + M\vec{L}
\end{aligned} \tag{2-12}$$

where M and N are linear integrodifferential operators on the tangential vector fields \vec{K} and \vec{L} over S .

Under any orthonormal coordinates (u, v) over S having \hat{u} , \hat{v} as the unit basis vectors and with $\hat{n} = \hat{u} \times \hat{v}$, a tangential vector field \vec{A} over S can be written in matrix form as:

$\vec{A} = \begin{bmatrix} A_u \\ A_v \end{bmatrix}$. Then $\hat{n} \times \vec{A} = \begin{bmatrix} -A_v \\ A_u \end{bmatrix} = -i\sigma_2 \vec{A}$ where $\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ is one of the Pauli spin

matrices. Note that $\sigma_2^2 = 1$. Using these matrix notations, we can rewrite Eq. (2-12) in the following form:

$$\begin{bmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{bmatrix} \begin{bmatrix} \vec{K}^+ - \vec{K}^- \\ \vec{L}^+ - \vec{L}^- \end{bmatrix} = \begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} \vec{K} \\ \vec{L} \end{bmatrix} - 2 \begin{bmatrix} \vec{E}_{\tan}^{inc} \\ \vec{H}_{\tan}^{inc} \end{bmatrix} \tag{2-13}$$

C. IMPEDANCE BOUNDARY CONDITION

Maxwell equations alone cannot determine the electromagnetic fields completely. If S is an open surface, appropriate boundary condition which the fields satisfy on S must be specified. It is usually given in terms of the impedance boundary condition, a linear relation among the tangential components of the total \vec{E} and the total \vec{H} fields on S . If S is a closed surface, there are two possibilities: One is to specify the electric and magnetic properties of

the volume within S and require the fields to satisfy regularity conditions within S and be linked to the fields outside through boundary conditions across S ; another is to specify the impedance boundary condition on S^+ for an exterior problem or on S^- for an interior problem. Note that an impedance boundary condition over a closed surface S completely separates the exterior from the interior of S . Therefore, the surface impedance on S^- can be arbitrary for an exterior problem while that on S^+ can be arbitrary for an interior problem. In this thesis, normalized surface impedances Z^\pm are assumed to be specified on S^\pm whether S is an open or a closed surface. Note that an open surface S can be considered as bounded within the closed surface formed by joining S^+ and S^- .

The impedance boundary conditions on S^\pm are defined by:

$$\hat{n}^\pm \times (\vec{E}^\pm \times \vec{n}^\pm) = Z^\pm (\hat{n}^\pm \times \vec{H}^\pm) \quad (2-14)$$

or equivalently, in terms of the total surface currents:

$$\hat{n}^\pm \times \vec{L}^\pm = Z^\pm \vec{K}^\pm \quad (2-15)$$

With the matrix notations for tangential vector fields over S in the orthonormal coordinate system (u, v) introduced before, we can consider Z^\pm as 2×2 matrices and rewrite eq. (2-15) in the form:

$$\mp i\sigma_2 \vec{L}^\pm = Z^\pm \vec{K}^\pm = \frac{1}{2} Z^\pm [\vec{K}^+ - (\vec{K}^+ - \vec{K}^-)] \quad (2-16)$$

which can readily be recast into a relation among sum and difference currents:

$$\begin{bmatrix} -\Delta & -i\sigma_2 \\ Z & 0 \end{bmatrix} \begin{bmatrix} \vec{K}^+ - \vec{K}^- \\ \vec{L}^+ - \vec{L}^- \end{bmatrix} = \begin{bmatrix} Z & 0 \\ -\Delta & -i\sigma_2 \end{bmatrix} \begin{bmatrix} \vec{K} \\ \vec{L} \end{bmatrix} \quad (2-17)$$

with

$$Z = \frac{1}{2} (Z^+ + Z^-) \quad (2-18)$$

and

$$\Delta = \frac{1}{2} (Z^+ - Z^-) \quad (2-19)$$

Eqs. (2-13) and (2-17) are a set of four two-dimensional vector equations to be solved for the sum and difference equivalent electric and magnetic surface current densities on S .

D. ALGEBRA OF THE SUM-DIFFERENCE CURRENT EQUATIONS

The existence and uniqueness of solution to either the exterior or the interior problem specified in terms of the impedance boundary condition have been well established [3]. Here we want to investigate how such a solution can be obtained from eqs. (2-13) and (2-17). Clearly eq. (2-17) defines uniquely the algebraic relationship between the difference and the sum currents if Z is invertible. For example, the difference currents can be expressed in terms of the sum currents:

$$\begin{bmatrix} \vec{K}^+ - \vec{K}^- \\ \vec{L}^+ - \vec{L}^- \end{bmatrix} = - \begin{bmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{bmatrix} R \begin{bmatrix} \vec{K} \\ \vec{L} \end{bmatrix} \quad (2-20)$$

where

$$R = \begin{bmatrix} Z - \Delta Z^{-1} \Delta & -i\Delta Z^{-1} \sigma_2 \\ -i\sigma_2 Z^{-1} \Delta & \sigma_2 Z^{-1} \sigma_2 \end{bmatrix} \quad (2-21)$$

An equation for the sum currents is obtained by substituting eq. (2-20) into eq. (2-13):

$$\begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} \vec{K} \\ \vec{L} \end{bmatrix} + R \begin{bmatrix} \vec{K} \\ \vec{L} \end{bmatrix} = 2 \begin{bmatrix} \vec{E}_{\tan}^{inc} \\ \vec{H}_{\tan}^{inc} \end{bmatrix} \quad (2-22)$$

which can be solved for \vec{K} and \vec{L} . Eq. (2-20) in turn enables us to compute the difference currents algebraically then split the sum and the difference currents into total currents on S^\pm .

If Z is not invertible, then the situation is more complicated. Z can either be of rank 0 when $Z = 0$ or rank 1 when $\det[Z] = 0$ but $Z \neq 0$. Eqs. (2-13) and (2-17) can be combined into an equation for the sum currents \vec{K} and \vec{L} :

$$\begin{bmatrix} 1 & i\Delta\sigma_2 \\ 0 & -iZ\sigma_2 \end{bmatrix} \begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} \vec{K} \\ \vec{L} \end{bmatrix} + \begin{bmatrix} Z & 0 \\ -\Delta & -i\sigma_2 \end{bmatrix} \begin{bmatrix} \vec{K} \\ \vec{L} \end{bmatrix} = 2 \begin{bmatrix} 1 & i\Delta\sigma_2 \\ 0 & -iZ\sigma_2 \end{bmatrix} \begin{bmatrix} \vec{E}_{\tan}^{inc} \\ \vec{H}_{\tan}^{inc} \end{bmatrix} \quad (2-23)$$

When $Z = 0$, eq. (2-17) gives the null relations $\vec{L}^+ - \vec{L}^- = i\sigma_2\Delta(\vec{K}^+ - \vec{K}^-)$ and $\vec{L} = i\sigma_2\Delta\vec{K}$ hence $\vec{L}^\pm = i\sigma_2\Delta\vec{K}^\pm$. Eq. (2-23) becomes one for \vec{K} only:

$$\begin{bmatrix} 1 & i\Delta\sigma_2 \end{bmatrix} \begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} 1 \\ i\sigma_2\Delta \end{bmatrix} \vec{K} = 2 \begin{bmatrix} 1 & i\Delta\sigma_2 \end{bmatrix} \begin{bmatrix} \vec{E}_{\tan}^{inc} \\ \vec{H}_{\tan}^{inc} \end{bmatrix} \quad (2-24)$$

Eq. (2-13) has to be used to find the difference electric current:

$$\vec{K}^+ - \vec{K}^- = i\sigma_2 [N + iM\sigma_2\Delta] \vec{K} - 2i\sigma_2 \vec{H}_{\tan}^{inc} \quad (2-25)$$

Therefore,

$$\vec{K}^{\pm} = \frac{1}{2} \{ 1 \pm i\sigma_2 [N + iM\sigma_2\Delta] \} \vec{K} \mp i\sigma_2 \vec{H}_{\tan}^{inc} \quad (2-26)$$

Since the last term in eq. (2-26) is $\vec{K}^{\pm,inc}$,

$$\vec{K}^{\pm,sc} = \frac{1}{2} \{ 1 \pm i\sigma_2 [N + iM\sigma_2\Delta] \} \vec{K} \quad (2-27)$$

\vec{L}^+ and \vec{L}^- can be obtained algebraically by multiplying $i\sigma_2\Delta$ to \vec{K}^+ and \vec{K}^- respectively.

On the other hand, by eq. (2-13),

$$\vec{L}^{\pm,sc} = \frac{1}{2} i\sigma_2 \{ \Delta \mp [M - iN\sigma_2\Delta] \} \vec{K} \quad (2-28)$$

Note that the $Z = 0$ case includes the special situation $Z^+ = Z^- = Z = \Delta = 0$ when both sides of S are perfectly conducting. Under this circumstance $\vec{L} = \vec{L}^{\pm} = 0$ and the operator N is never involved.

When Z is rank 1, $Z \neq 0$ but $\det[Z] = 0$. The right hand side of eq. (2-17) provides one linear relation between the components of $\vec{L} - i\sigma_2\Delta\vec{K}$ which can be used to reduce the four components of \vec{K} and \vec{L} as the unknown quantities in eq. (2-23) to three so that the remaining three components can be solved. The left hand side of eq. (2-17) assures that the same linear relation between the components of $\vec{L} - i\sigma_2\Delta\vec{K}$ exists between corresponding components of the difference currents. Eq. (2-13) again has to be invoked to compute three other linearly independent combinations of the components of the difference currents from the sum currents.

E. CONSIDERATIONS FOR A CLOSED SURFACE

When S is a closed surface, the choice of Z^- can be arbitrary for an exterior problem such as scattering while the choice of Z^+ is arbitrary for an interior problem. It is desirable to choose $Z^- = -Z^+$ so that $Z = 0$ and $\Delta = Z^+ = -Z^-$. Then we have $\vec{L} = i\sigma_2\Delta\vec{K}$ and only \vec{K} has to be computed. With such a choice, for an exterior problem,

$$\vec{K}^{+,sc} = \frac{1}{2}(1 - \sigma_2 M \sigma_2 Z^+ + i \sigma_2 N) \vec{K} \quad (2-29)$$

$$\vec{L}^{+,sc} = \frac{i\sigma_2}{2}(Z^+ + i N \sigma_2 Z^+ - M) \vec{K} \quad (2-30)$$

and for an interior problem:

$$\vec{K}^{-,sc} = \frac{1}{2}(1 - \sigma_2 M \sigma_2 Z^- - i \sigma_2 N) \vec{K} \quad (2-31)$$

$$\vec{L}^{-,sc} = \frac{i\sigma_2}{2}(M - Z^- + i N \sigma_2 Z^-) \vec{K} \quad (2-32)$$

III. A THEOREM OF ANISOTROPIC ABSORBERS

A. AXIAL RADIATION FROM A SURFACE OF 90° ROTATIONAL SYMMETRY

The xy -plane cross section of a surface S having a 90° rotational symmetry around the z -axis is shown in Figure 3-1. Because of this symmetry, S can be decomposed into four non-overlapping congruent pieces S_1, S_2, S_3, S_4 so that a 90° rotation will bring S_i into S_{i+1} .

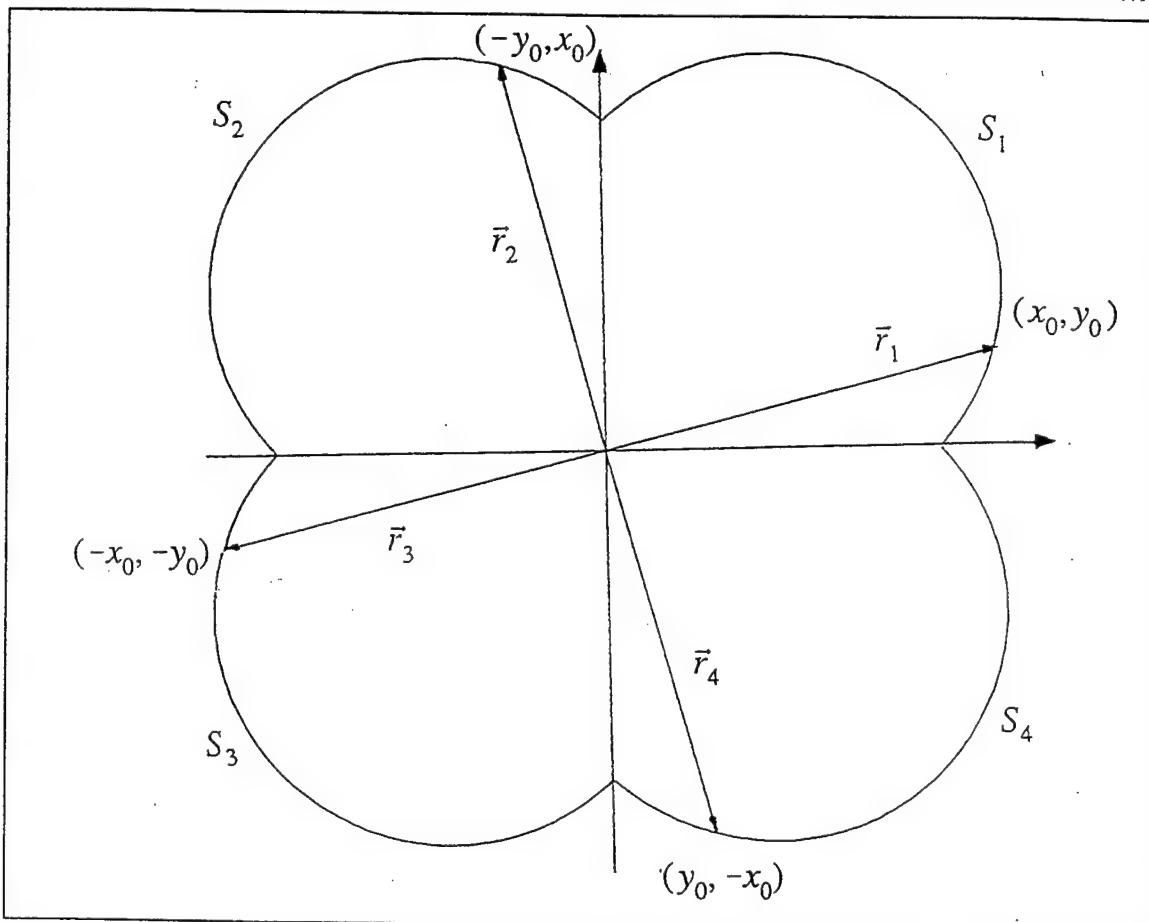


Figure 3-1 Cross Section of a Surface of 90-Degree Rotational Symmetry.

(These subscripts are considered as equal under modulo 4.) Therefore each piece S_i of S can

be parametrized in the same orthonormal coordinates (u, v) , with $\hat{u} = \frac{\partial \vec{r}_i}{\partial u}$, $\hat{v} = \frac{\partial \vec{r}_i}{\partial v}$ the orthonormal basis vectors on S_i , as follows:

$$\begin{aligned}\vec{r}_1 &= (x_o(u, v), y_o(u, v), z_o(u, v)) \\ \vec{r}_2 &= (-y_o(u, v), x_o(u, v), z_o(u, v)) \\ \vec{r}_3 &= (-x_o(u, v), -y_o(u, v), z_o(u, v)) \\ \vec{r}_4 &= (y_o(u, v), -x_o(u, v), z_o(u, v))\end{aligned}\tag{3-1}$$

where $\vec{r}_i \in S_i$. As an example of a possible choice, $u = \text{constant}$ and $v = \text{constant}$ can be the lines of curvature of S_i .

In terms of the coordinates (u, v) , the sum surface current distributions on S_i are:

$$\begin{aligned}\vec{K}(\vec{r}_i) &= \vec{K}_i(u, v) \\ \vec{L}(\vec{r}_i) &= \vec{L}_i(u, v)\end{aligned}\tag{3-2}$$

Since for $r \gg r_o$,

$$G(\vec{r}) \approx \frac{e^{ik|\vec{r}-\vec{r}_o|}}{k r}\tag{3-3}$$

$$\nabla G(\vec{r}) \approx ik\hat{r}G \approx -\nabla_o G(\vec{r})\tag{3-4}$$

the radiation from such current distributions at a distance $r \gg \max |\vec{r}_i|$ along the positive z-axis is, from eq. (2-6):

$$\begin{aligned}
\vec{E}^{sc}(\vec{r}) &= \vec{E}_{s-c}(\vec{r}, S, \vec{K}, \vec{L}) \\
&= \frac{ik}{4\pi r} \int_S \left\{ \hat{x} [K_x(\vec{r}_o) + L_y(\vec{r}_o)] + \hat{y} [K_y(\vec{r}_o) - L_x(\vec{r}_o)] \right\} e^{ik\sqrt{(z-z_o)^2 + x_o^2 + y_o^2}} da_o \\
&= \frac{ik}{4\pi r} \int \int_{S_1} \left\{ \hat{x} \sum_{i=1}^4 [K_{ix}(u, v) + L_{iy}(u, v)] \right. \\
&\quad \left. + \hat{y} \sum_{i=1}^4 [K_{iy}(u, v) - L_{ix}(u, v)] \right\} e^{ik\sqrt{(z-z_o)^2 + x_o^2 + y_o^2}} du dv
\end{aligned} \tag{3-5}$$

Note that this approximation is independent of the wavelength; it is applicable in regions closer to S than the usual Fresnel zone.

B. CONDITION FOR VANISHING ON-AXIS BACKSCATTERING

Consider two situations when a linearly polarized plane wave of unit strength is incident on S along the z -axis from the positive direction: Situation 1, identified with the superscript (1) has the wave polarized in the x -direction while Situation 2, identified with the superscript (2), has the wave polarized in the y -direction. The incident fields are respectively:

$$\begin{aligned}
\vec{E}^{inc,(1)} &= \hat{x} e^{-ikz} \\
\vec{H}^{inc,(1)} &= -\hat{y} e^{-ikz}
\end{aligned} \tag{3-6}$$

$$\begin{aligned}
\vec{E}^{inc,(2)} &= \hat{y} e^{-ikz} \\
\vec{H}^{inc,(2)} &= \hat{x} e^{-ikz}
\end{aligned} \tag{3-7}$$

Note that, as seen from the positive z -axis, the incident wave in Situation 2 is that of Situation 1 rotated by 90° counterclockwise. Furthermore, Situation 2 can be obtained from Situation 1 through the duality transformation $\vec{E}^{inc} \rightarrow \vec{H}^{inc}$, $\vec{H}^{inc} \rightarrow -\vec{E}^{inc}$. Therefore, for a plane wave, a 90° rotation is equivalent to undergoing the duality transform.

Because of the rotational symmetry of S , the currents excited on S_i under Situation 1 must appear on S_{i+1} under Situation 2. Therefore:

$$\begin{aligned}
K_{i+1,x}^{(2)}(u, v) &= -K_{i,y}^{(1)}(u, v) \\
K_{i+1,y}^{(2)}(u, v) &= K_{i,x}^{(1)}(u, v) \\
K_{i+1,z}^{(2)}(u, v) &= K_{i,z}^{(1)}(u, v)
\end{aligned} \tag{3-8}$$

$$\begin{aligned}
L_{i+1,x}^{(2)}(u, v) &= -L_{i,y}^{(1)}(u, v) \\
L_{i+1,y}^{(2)}(u, v) &= L_{i,x}^{(1)}(u, v) \\
L_{i+1,z}^{(2)}(u, v) &= L_{i,z}^{(1)}(u, v)
\end{aligned} \tag{3-9}$$

Assume that Z on S is invertible, the sum currents on S are determined by Eq. (2-22). The tangential components of the incident fields which appear on the right-hand-side of that equation under these two situations are:

$$\begin{bmatrix} \vec{E}_{\tan}^{inc,(1)} \\ \vec{H}_{\tan}^{inc,(1)} \end{bmatrix} = \begin{bmatrix} \hat{u} \cdot \hat{x} \\ \hat{v} \cdot \hat{x} \\ -\hat{u} \cdot \hat{y} \\ -\hat{v} \cdot \hat{y} \end{bmatrix} e^{-ikz} \tag{3-10}$$

$$\begin{bmatrix} \vec{E}_{\tan}^{inc,(2)} \\ \vec{H}_{\tan}^{inc,(2)} \end{bmatrix} = \begin{bmatrix} \hat{u} \cdot \hat{y} \\ \hat{v} \cdot \hat{y} \\ \hat{u} \cdot \hat{x} \\ \hat{v} \cdot \hat{x} \end{bmatrix} e^{-ikz} = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \vec{E}_{\tan}^{inc,(1)} \\ \vec{H}_{\tan}^{inc,(1)} \end{bmatrix} \tag{3-11}$$

Therefore,

$$\begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} \vec{K}^{(1)} \\ \vec{L}^{(1)} \end{bmatrix} + R \begin{bmatrix} \vec{K}^{(1)} \\ \vec{L}^{(1)} \end{bmatrix} = 2 \begin{bmatrix} \vec{E}_{\tan}^{inc,(1)} \\ \vec{H}_{\tan}^{inc,(1)} \end{bmatrix} \quad (3-12)$$

$$\begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} \vec{K}^{(2)} \\ \vec{L}^{(2)} \end{bmatrix} + R \begin{bmatrix} \vec{K}^{(2)} \\ \vec{L}^{(2)} \end{bmatrix} = 2 \begin{bmatrix} \vec{E}_{\tan}^{inc,(2)} \\ \vec{H}_{\tan}^{inc,(2)} \end{bmatrix} = 2 \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \vec{E}_{\tan}^{inc,(1)} \\ \vec{H}_{\tan}^{inc,(1)} \end{bmatrix} \quad (3-13)$$

Since

$$\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} M & -N \\ N & M \end{bmatrix} = \begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \quad (3-14)$$

it follows that if

$$\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} R = R \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \quad (3-15)$$

we can multiply $\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$ to Eq. (3-12) to get:

$$\begin{bmatrix} M & -N \\ N & M \end{bmatrix} \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \vec{K}^{(1)} \\ \vec{L}^{(1)} \end{bmatrix} + R \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \vec{K}^{(1)} \\ \vec{L}^{(1)} \end{bmatrix} = 2 \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \vec{E}_{\tan}^{inc,(1)} \\ \vec{H}_{\tan}^{inc,(1)} \end{bmatrix} \quad (3-16)$$

Therefore the excited surface currents in these two situations are related by:

$$\begin{bmatrix} \vec{K}^{(2)} \\ \vec{L}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} \vec{K}^{(1)} \\ \vec{L}^{(1)} \end{bmatrix} = \begin{bmatrix} -\vec{L}^{(1)} \\ \vec{K}^{(1)} \end{bmatrix} \quad (3-17)$$

Combining this result with Eqs. (3-8) and (3-9), we have:

$$\begin{aligned}
 K_{i+1,x}^{(2)}(u, v) &= -L_{i+1,x}^{(1)}(u, v) = -K_{i,y}^{(1)}(u, v) \\
 K_{i+1,y}^{(2)}(u, v) &= -L_{i+1,y}^{(1)}(u, v) = K_{i,x}^{(1)}(u, v) \\
 L_{i+1,x}^{(2)}(u, v) &= K_{i+1,x}^{(1)}(u, v) = -L_{i,y}^{(1)}(u, v) \\
 L_{i+1,y}^{(2)}(u, v) &= K_{i+1,y}^{(1)}(u, v) = L_{i,x}^{(1)}(u, v)
 \end{aligned} \tag{3-18}$$

so that

$$\sum_{i=1}^4 [K_{i,x}^{(1)}(u, v) + L_{i,y}^{(1)}(u, v)] = 0 \tag{3-19}$$

and

$$\sum_{i=1}^4 [K_{i,y}^{(1)}(u, v) - L_{i,x}^{(1)}(u, v)] = 0 \tag{3-20}$$

Hence, only the positive z-axis, from Eq (3-5),

$$\begin{aligned}
 \vec{E}^{sc}(\vec{r}) &= \frac{ik}{4\pi r} \int \int_S \left\{ \hat{x} \sum_{i=1}^4 [K_{i,x}^{(1)}(u, v) + L_{i,y}^{(1)}(u, v)] \right. \\
 &\quad \left. + \hat{y} \sum_{i=1}^4 [K_{i,y}^{(1)}(u, v) - L_{i,x}^{(1)}(u, v)] \right\} e^{ik\sqrt{(z-z_o)^2 + x_o^2 + y_o^2}} du dv \\
 &= 0
 \end{aligned} \tag{3-21}$$

and the backscattering from S along the positive z-direction must vanish if Eq. (3-15) is satisfied.

C. IMPEDANCE MATRICES FOR ZERO ON-AXIS BACKSCATTERING

It can be verified that the matrix $\begin{bmatrix} Z - \Delta Z^{-1} \Delta & -i \Delta Z^{-1} \sigma_2 \\ -i \sigma_2 Z^{-1} \Delta & \sigma_2 Z^{-1} \sigma_2 \end{bmatrix}$ commutes with $\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$,

if and only if:

$$\sigma_2 Z^{-1} \Delta = -\Delta Z^{-1} \sigma_2 \quad (3-22)$$

$$Z - \sigma_2 Z^{-1} \sigma_2 = \Delta Z^{-1} \Delta \quad (3-23)$$

where both Z and Δ are 2×2 matrices. Under the assumption that the inverse of Z exists, we analyze Eqs. (3-22) and (3-23) as follows:

Because of the identity:

$$Z^{-1} = \frac{1}{\det Z} \sigma_2 Z^T \sigma_2 \quad (3-24)$$

and, by multiplying σ_2 to both sides of Eq. (3-22):

$$Z^{-1} \Delta = -\sigma_2 \Delta Z^{-1} \sigma_2 \quad (3-25)$$

Eq. (3-23) can be transformed to

$$\begin{aligned} Z &= -\Delta (\sigma_2 \Delta Z^{-1} \sigma_2) + \sigma_2 Z^{-1} \sigma_2 \\ &= -\Delta \sigma_2 \Delta \sigma_2 (\sigma_2 Z^{-1} \sigma_2) + \sigma_2 Z^{-1} \sigma_2 \\ &= \frac{1}{\det Z} [1 - (\Delta \sigma_2)^2] Z^T \end{aligned} \quad (3-26)$$

where

$$(\Delta \sigma_2)^2 = \Delta \sigma_2 \Delta \sigma_2 = \begin{bmatrix} \Delta_{11} \Delta_{22} - \Delta_{12}^2 & 0 \\ 0 & \Delta_{11} \Delta_{22} - \Delta_{21}^2 \end{bmatrix} \quad (3-27)$$

It is observed that Eq. (3-27) is general simplified if Δ is symmetric. Therefore, we consider only the case when $\Delta = \Delta^T$ then $\Delta_{12} = \Delta_{21}$ so that:

$$(\Delta \sigma_2)^2 = (\det \Delta) I \quad (3-28)$$

From Eq. (3-26),

$$Z = \left(\frac{1 - \det Z}{\det Z} \right) Z^T \quad (3-29)$$

Therefore, there are two cases:

Case I

$$Z = -Z^T = \begin{bmatrix} 0 & z_{12} \\ -z_{12} & 0 \end{bmatrix}, z_{12} \neq 0 \quad (3-30)$$

$$\det \Delta = 1 + \det Z = 1 + z_{12}^2 \quad (3-31)$$

Case II

$$Z = Z^T \quad (3-32)$$

$$\det Z + \det \Delta = 1 \quad (3-33)$$

On the other hand, substituting Eq. (3-24) into Eq. (3-22) yields:

$$z_{11}(\Delta_{12} - \Delta_{21}) = (z_{12} - z_{21}) \Delta_{11} \quad (3-34)$$

$$z_{22}(\Delta_{12} - \Delta_{21}) = (z_{12} - z_{21}) \Delta_{22} \quad (3-35)$$

$$z_{11}\Delta_{22} + z_{22}\Delta_{11} = 2z_{21}\Delta_{12} = 2z_{12}\Delta_{21} \quad (3-36)$$

For Case 1, Eqs (3-34) and (3-35) require that $\Delta_{11} = \Delta_{22} = 0$, Eq. (3-36) requires that $\Delta_{12} = \Delta_{21} = 0$. Therefore $\Delta = 0$. From Eq (3-31), $1 + z_{12}^2 = 0$. Therefore, $z_{12} = \pm i$. So that $Z^+ = Z^- = Z = \pm \sigma_2$.

For Case II, Eqs. (3-34) and (3-35) are trivially satisfied. Eq (3-36) becomes:

$$z_{11}\Delta_{22} + z_{22}\Delta_{11} - 2z_{12}\Delta_{12} = 0 \quad (3-37)$$

Eq. (3-33) is explicitly:

$$z_{11}z_{22} - z_{12}^2 + \Delta_{11}\Delta_{22} - \Delta_{12}^2 = 1 \quad (3-38)$$

Therefore, Eqs. (3-22) and (3-23) are satisfied if

$$Z^+ = Z^- = \pm \sigma_2 \quad (3-39)$$

or Z^+ and Z^- are symmetric and

$$\det Z^+ = 1 \quad (3-40)$$

$$\det Z^- = 1 \quad (3-41)$$

It should be noted that, if the shell is a closed surface, then the impedance boundary condition closes off the inside of the shell from outside. Therefore, either Z^+ is symmetric with $\det Z^+ = 1$ or $Z^+ = \pm \sigma_2$ is sufficient to eliminate on-axis backscattering. This is an extension of Weston's work [4] to anisotropic impedance coated body of revolution.

IV. CONCLUSIONS

In this report, we introduce the sum-difference surface current formulation of electromagnetic boundary value problems when impedances are specified on both sides of a surface. For an impedance coated body, the body can be treated as being a surface separating the space into two regions of identical medium. For an exterior problem, the impedance normalized to the medium on the inside surface, Z^- can be chosen arbitrarily; and for an interior problem, that on the outside surface, Z^+ , can be arbitrary. The choice when $Z^- = -Z^+$ is of particular interest because the integrodifferential equation has only the sum of the equivalent electric surface currents on the outside and the inside surfaces as its unknown to be solved.

This formulation preserves the duality nature of Maxwell equations and carries it over into the algebraic form of the integrodifferential operators in the equations for the sum currents. Since a 90° rotation is equivalent to undergoing a duality transform for an incident plane wave, this particular symmetry in the algebraic form of the operators leads to the sufficient conditions that if $Z^+ = Z^- = \pm \sigma_2$, or if Z^+ and Z^- are symmetric and $\det Z^+ = \det Z^- = 1$, the on-axis backscattering of an anisotropic impedance coated scatterer having a 90° rotational symmetry will be eliminated. Note that here Z^+ and Z^- may vary with location. This is an extension of Western's result [4] of which the surface impedance is isotropic.

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